Modeling the Spread of Disease - Influenza

By: Antonella Basso and Tisha Naipaul

Introduction:

The following paper provides a detailed outline of an explorative mathematical modeling project. The aim of this group project was to use Python to explore the SIR Model, which W. O. Kermack and A. G. McKendrick developed in 1927 (Kermack and McKendrick 1927), and provides a simulation of the following:

(I) Consider the famous case mentioned in a 1978 British Medical Journal article, which reported the spread of Influenza in a boys' boarding school (closed environment) for this example:

- There are 763 boys in a boarding school
- Assume there are no births, deaths, immigration, or emigration during the time of the simulation.
- On the first day of the simulation, only 1 boy has the flu, which is unfamiliar to the other boys.
- The epidemic lasts 14 days
- There are 56 total time steps as the simulation is looked at every ¼ day

To model the spread of the disease, over the 14 days, the SIR Model is used, which precisely considers the following three populations:

- **Susceptibles** (S): have no immunity from the disease.
- **Infecteds** (I): have the disease and can spread it to others.
- **Recovereds** (R): have recovered from the disease and are immune to further infection.

Having been given the preliminary code for the simulation, the goal of the project was to gain a deeper understanding of the SIR Model and to observe the roles that other factors play in the spread of the disease by modifying the code to respond to the following:

1. Adjust the SIR model to allow for vaccination of susceptible boys. Assume that 15% are vaccinated each day, and make a simplifying assumption that immunization begins immediately. Discuss the effect on the duration and intensity of the epidemic. Consider the impact of other vaccination rates.

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NCSLP. "Modeling the Spread of SARS-Containing Emerging Disease." *Introduction to Computational Science: Modeling and Simulation for the Sciences*, by Angela B. Shiflet and George W. Shiflet, 2nd ed., Princeton University Press, 2014, pp. 131–146.

2. Adjust the SIR model to allow for vaccination of susceptible boys. Assume that 15% are vaccinated each day and that immunization begins after 3 days. Discuss the effect on the duration and intensity of the epidemic. Consider the impact of other vaccination rates.
3. Adjust the SIR model to allow for vaccination of susceptible boys. Assume that all children are vaccinated 2 days before a boy comes down with the flu and that immunization begins after 4 days. Discuss the effect on the duration and intensity of the epidemic. (I)

In addition to responding to these questions, we were asked to consider the impact of other vaccination rates to further explore the relationship between the number of boys vaccinated and the number of boys that contract the disease. In other words, we were asked to analyse the effectiveness of taking preventative measures (immunizations). To explore this relationship we seeked to answer the following:

- How would changing the percentage of boys vaccinated per day from 15% to 7.5% and 30% affect the total number of susceptibles, infecteds and recovereds at the end of the 14 days if the vaccines were effective immediately (as in question 1)?
- How would changing the time it takes for the vaccination to work from 4 days to 8 days affect the spread of the flu when all the boys are vaccinated 2 days before the start of the simulation (as in question 3)?

Explanation of the Mathematics:
In order to predict the spread of disease, we must understand the math behind an SIR model. The **S**usceptible boys are the boys that have a chance of catching the disease. We calculate their growth rate with the equation

$$\frac{dS}{dt} = -\left(\frac{kb}{N}\right)SI\,.$$

The k is the number of contacts a boy will make in a day, the b is the chance of catching the disease, and the N is the total population of boys. This multiplier is commonly referred to as r. It is then multiplied by the number of susceptible boys and the number of infected boys. In this case, we are assuming that the boys contact an average of 33.3 boys per day, and there is a 5% chance of catching the flu per encounter. Then our infection rate (r) would equal 0.00218. The **R**ecovered boys are the boys who have had the disease and have recovered from it. To calculate the growth rate of the recovered boys is much simpler since it is proportional to the number of infected boys. We use the equation

$$\frac{dR}{dt} = aI$$

where a is the rate at which the boys recover and the I is the number of infected boys. In this case, we are assuming that the boys are sick with the flu for an average of 2 days. This means that $a = ½$ (t in days). With this information we can calculate the growth rate of the **I**nfected boys. The infected boys are the susceptible boys who caught the disease. We calculate their growth rate with the equation

$$\frac{dI}{dt} = -\frac{dS}{dt} - \frac{dR}{dt}.$$

With this basic concept of an SIR model and the Python code to accompany it, we were able to build upon it and model an SVIR model.

Code:

The initial Python code that we were given follows the basic assumptions that were discussed in the introduction. We are dealing with a closed environment with no births and no deaths. We start the code by declaring these assumptions based on the SIR math that was explained previously.

```
numIterations = int(simLength/DT) + 1
t = 0

susceptibles = 762 #number of boys who aren't sick
infecteds = 1 #number of sick boys
recovereds = 0 #number of boys who recovered from the flu

infection_rate = 0.00218 #rate at which the boys are infected
recovery_rate = 0.5 #rate at which the boys recover

get_sick = infection_rate * susceptibles * infecteds #number of boys who get sick
recover = recovery_rate * infecteds #number of boys who recover
```

The first line calculates the number of iterations the code will run through which is based on the total number of days (simLength = 14) and the Δt (DT = 0.25) in days. We are starting our model at t = 0. The next part assigns a value to the variable 'susceptibles'. This variable starts at 762 which is the number of boys at the school who have not had the flu.One boy came to school with the flu, which is represented by the variable 'infecteds', and no boys have recovered from the flu at the beginning of our model, which is the variable 'recovereds'. The next declaration is the value of the infection rate (*r*), followed by the value of the recovery rate (*a*). These values are used to calculate the growth rate of susceptible boys (get_sick) and the growth rate of the recovered boys (recover). We use the equation for 'get_sick' from the dS/dt equation and the equation for 'recover' from the dR/dt equation. With these basic assumptions we can move into our **for** loop to start calculating the spread of disease.

NCSLP (National Computational Science Leadership Program). "Influenza Epidemic in a Boarding School." The Shodor Educational Foundation, Inc. Original from J. D. Murray, Mathematical Biology, New York: Springer-Verlag, 1989. http://www.shodor.org/ncslp/talks/basicstella/sld009.htm (accessed November 17, 2012)

```
for i in range(1, numIterations):
    t = i * DT #calculates the time t in days
    susceptibles = susceptibles + (-get_sick) * DT #calculates the number of boys susceptible
    #to the flu every 6 hours
    infecteds = infecteds + (get_sick - recover) * DT #calculates the number of infected boys
    #every 6 hours
    recovereds = recovereds + (recover) * DT #calculates the number of recovered boys

    get_sick = infection_rate * susceptibles * infecteds
    recover = recovery_rate * infecteds
```

The **for** loop runs through the number of iterations (based on DT and simLength) which is every 6 hours in this case. The first line in the loop starts calculating the time t in days. This will be useful when we start graphing. Then we start calculating the growth of the susceptible population. This growth rate should be negative since we are assuming that there is no way for the total population to increase and the recovered boys can't become susceptible again, so the rate 'get_sick' is negative. We add this rate to the previous number of susceptible boys and then multiply the whole equation by Δt to get the number of boys who haven't gotten the flu yet. The next equation calculates the number of boys infected by the flu in the last 6 hours. This rate is determined by the rate of boys getting sick (get_sick) - the rate of the boys who are recovering (recover). This rate is added to the number of boys already infected and then multiplied by Δt to get the number of boys who have gotten the flu. The next equation calculates the number of boys who have recovered from the flu in the last 6 hours, which is the number of boys already recovered added to the rate the boys are recovering, and then multiplied by the Δt. The last two equations update the rate at which the boys are getting sick and the rate at which they are recovering based on the new values of SIR which we just calculated. We add these populations to lists and put them in a dataframe so that we can create a graph and see the spread of disease.

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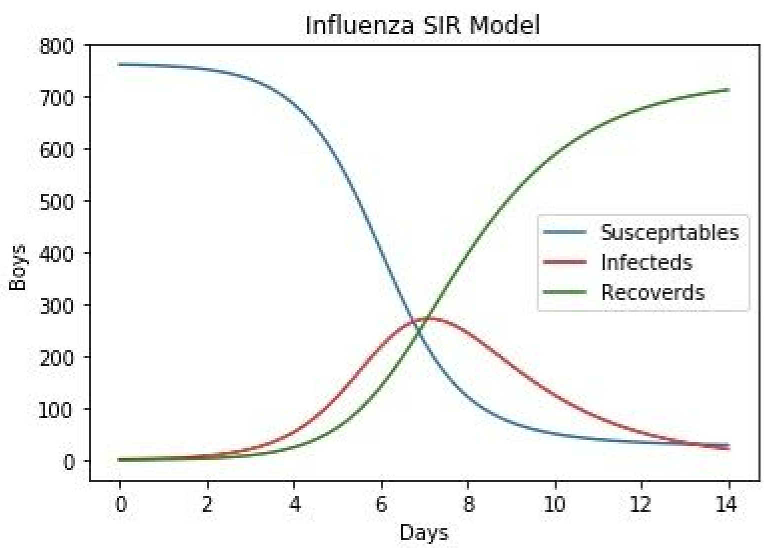
	Days	Susceprtables	Infecteds	Recoverds
0	0.0	762.000000	1.000000	0.000000
4	1.0	759.469082	2.768200	0.762718
8	2.0	752.517754	7.613190	2.869056
16	4.0	686.099797	52.995515	23.904688
24	6.0	405.010816	217.729193	140.259991
32	8.0	121.829316	245.171988	395.998696
40	10.0	50.355311	125.175595	587.469094
48	12.0	33.810097	52.729955	676.459948
56	14.0	28.768363	21.130682	713.100956



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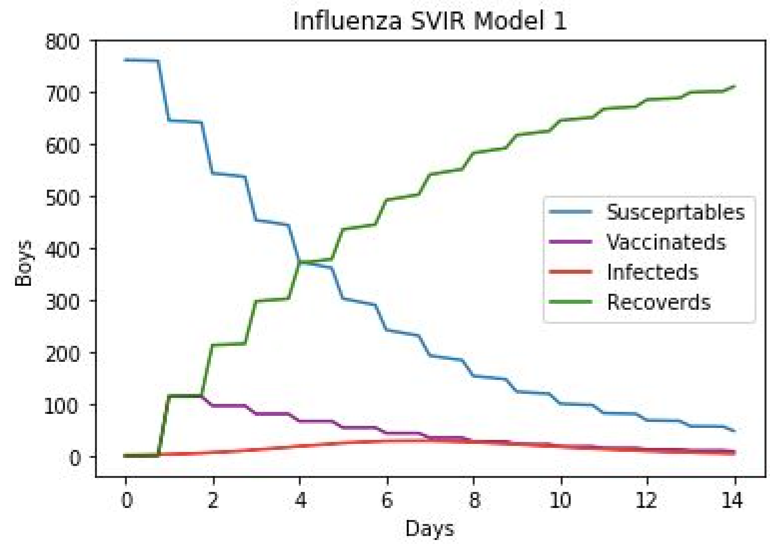
Questions:

(1) The first simulation we ran introduced a vaccine into the environment. We are assuming that 15% of the susceptible boys receive a flu vaccine every day starting on day 1. Once a boy is vaccinated, he moves immediately to the recovered population.

```
for i in range(1, numIterations): #runs the for loop based on delta t
    t = i * DT #calculating time

    if t % 1 == 0: #checks if the time is a whole number, this code will run every day
        vaccinated_boys = susceptibles * vac_percent #calcualtes the number of boys who will be vaccina
        susceptibles = susceptibles + (-get_sick) * DT - vaccinated_boys #calculates the numbers of sus
        infecteds = infecteds + (get_sick - recover) * DT #calculated the number of boys who caught the
        recovereds = recovereds + recover * DT + vaccinated_boys #calculates the number of boys who rec

    else:
        susceptibles = susceptibles + (-get_sick) * DT #calculates the number the number of susceptible
        infecteds = infecteds + (get_sick - recover) * DT #calculates the number of infected boys every
        recovereds = recovereds + (recover) * DT #calculates the number of recovered boys every 6 hours

    get_sick = infection_rate * susceptibles * infecteds #recalculated the number of boys getting sick
    recover = recovery_rate * infecteds #recalculates the number of boys recovering based on the new nu
```

We introduce an **if** statement into the **for** loop which determines whether or not a day has passed. A new variable has been added: 'vaccinated_boys'. This variable is the number of boys vaccinated each day. This is calculated by multiplying the number of susceptible boys by the percent of boys getting vaccinated (vac_percent), which is 0.15 in this case. We then subtract this number from the number of susceptible boys and add this number to the number of recovered boys every day. Otherwise, we run the code normally.

	Days	Susceprtables	Vaccinateds	Infecteds	Recoverds
0	0.0	762.000000	0.000000	1.000000	0.000000
4	1.0	645.415270	114.053812	2.768200	114.816530
8	2.0	543.735626	96.268354	6.250537	213.013837
16	4.0	373.399901	66.597083	18.388781	371.211318
24	6.0	242.328359	43.530659	28.385403	492.286237
32	8.0	153.775020	27.615235	26.356017	582.868963
40	10.0	100.252149	17.903420	17.349825	645.398026
48	12.0	68.112682	12.097785	9.228848	685.658471
56	14.0	47.715292	8.446031	4.295769	710.988938



As we would expect from introducing a vaccine into a closed environment, it takes less time for the number of recovered boys to overcome the number of susceptible boys. In the original graph, we saw this change happen around the 7th day, while it occurs around the 5th day in this model. The difference between each day is to be expected as well, since the vaccine is administered every day. Hardly any boys actually get the flu, about 28 boys at most around the 7th day, so there is a lower chance of the susceptible boys getting infected as well. Thus, the number of recovered boys would be slightly lower in this model. One may be inclined to think that since the final number of recovereds is lower with the introduction of the vaccine that the vaccine is inefficient. However, it is important to note that the total amount of infecteds at the end of the simulation without the vaccine is significantly higher. Moreover, the number of susceptibles is higher in this model because as mentioned, the number of boys that actually get infected is lower. Therefore, the total number of boys without the flu is higher in the SVIR model than in the SIR model (that is, the combined number of susceptibles and recovereds).

(2) The second simulation we ran included a vaccine with a delay in the effectiveness. Just like the previous problem, a vaccine was administered every day to 15% of the susceptible boys. This time the vaccine starts working three days after it is administered. In our simulation, this means that if a boy is vaccinated on day 1, then his vaccine will start working at 12:00am on day 4.

```
susceptibles = 762 #number of boys who have not been vaccinated
infecteds = 1 #number of boys who have the flu
recovereds = 0 #number of boys who have recovered from the flu

infection_rate = 0.00218 #rate at which the boys get sick
recovery_rate = 0.5 #rate at which the boys recover
vac_percent = 0.15 #perctage of boys vaccinated per day

get_sick = infection_rate * susceptibles * infecteds #calculates the number of boys who get
recover = recovery_rate * infecteds #calculates the number of boys who recover
vaccinated_today = 0 #boys who are vaccinated at 12:00am today
vaccinated_yesterday = 0 #boys who were vaccinated yesterday
vaccinated_twodaysago = 0 #boys who were vaccinated 2 days ago
```

This question gave us the most trouble, since we decided not to incorporate a vaccination rate. All our vaccines start working immediately on the day that they are supposed to. We decided to incorporate 3 additional variables in our code in order to keep track of the number of boys who will recover on the 4th day and how many boys who were vaccinated get sick every day. The idea is that we have four groups: unvaccinated, vaccinated, infected, and recovered. If you are vaccinated, you have three choices: get infected, recover (if 3 days have passed), or move on and get added to the next group of vaccinated boys. If you are unvaccinated, you have two choices: get infected, or move on and a portion of you will get vaccinated the next day. The infected group is the number of boys who are vaccinated but still sick plus the number of boys who are unvaccinated and got sick. The susceptible group is just the number of boys who are unvaccinated since we account for the vaccinated boys getting sick later on in the code.

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```
f t % 1 == 0: #checks if t is a whole number, this code will run every day


  if t > 3: #if we are past the third day
      vaccinated_twodaysago = vaccinated_yesterday - (infection_rate*vaccinated_yesterday*in
      #updates the number of boys vaccinated 2 days ago, subtracting the num of boys who get
      s1 = (infection_rate*vaccinated_yesterday*infecteds)
      #num of boys who were vaccinated two days ago but got sick
      vaccinated_yesterday = vaccinated_today - (infection_rate*vaccinated_today*infecteds)
      #updates the number of boys vaccinated yesterday, subtracting the num of boys who got
      s2 = (infection_rate*vaccinated_today*infecteds)
      #num of boy who were vaccinated yesterday but got sick
      vaccinated_today = susceptibles * vac_percent
      #calculates num of boys who were vaccinated today
      susceptibles -= vaccinated_today
      #num of unvaccinated boys
      vaccinated_today -= (infection_rate * vaccinated_today * infecteds) * DT
      #updates num of boys who were vaccinated today, subtracting the num of boys who got si
      susceptibles = susceptibles + (-get_sick) * DT #calculates the num of susceptible boys
      infecteds = infecteds + ((get_sick+ s1+s2+(infection_rate * vaccinated_today * infecte
      recovereds = recovereds + recover * DT + vaccinated_twodaysago
      #calculates the num of recovered boys adding the num of boys who were vaccinated two d
```

In this section of the code, we have confirmed that three days have passed with an **if** statement. This means that the boys who were vaccinated three days ago will now be considered "recovered." First, we update the variables 'vaccinated_twodaysago', 'vaccinated_yesterday', and 'vaccinated_today' to account for the boys who got sick even though they were vaccinated. We update the number of boys vaccinated today, update the number of boys who still haven't been vaccinated, and then finally calculate the number of boys who recovered from the flu, including the remaining boy who were vaccinated two days ago.

```
elif t >= 1:
    vaccinated_twodaysago = vaccinated_yesterday - (infection_rate * vaccinated_yesterday*infecteds)*DT
    #updates the number of boys vaccinated 2 days ago, subtracting the num of boys who get sick
    s1 = (infection_rate * vaccinated_yesterday*infecteds)
    #num of boys who were vaccinated two days ago but got sick
    vaccinated_yesterday = vaccinated_today - (infection_rate * vaccinated_today * infecteds) *DT
    #updates the number of boys vaccinated yesterday, subtracting the num of boys who got sick
    s=(infection_rate * vaccinated_today * infecteds)
    #num of boy who were vaccinated yesterday but got sick
    vaccinated_today = susceptibles * vac_percent
    #calculates num of boys who were vaccinated today
    susceptibles -= vaccinated_today #num of unvaccinated boys
    vaccinated_today -= (infection_rate * vaccinated_today * infecteds) *DT
    #updates num of boys who were vaccinated today, subtracting the num of boys who got sick
    susceptibles = susceptibles + (-get_sick) * DT #calculated the num of susceptible boys
    infecteds = infecteds + ((get_sick+s+s1+(infection_rate * vaccinated_today * infecteds)) - recover) * DT
    #calculates the num of infected boys adding the num who were vaccinated but still got sick
    recovereds = recovereds + recover * DT #calculates the num of recovered boys
```


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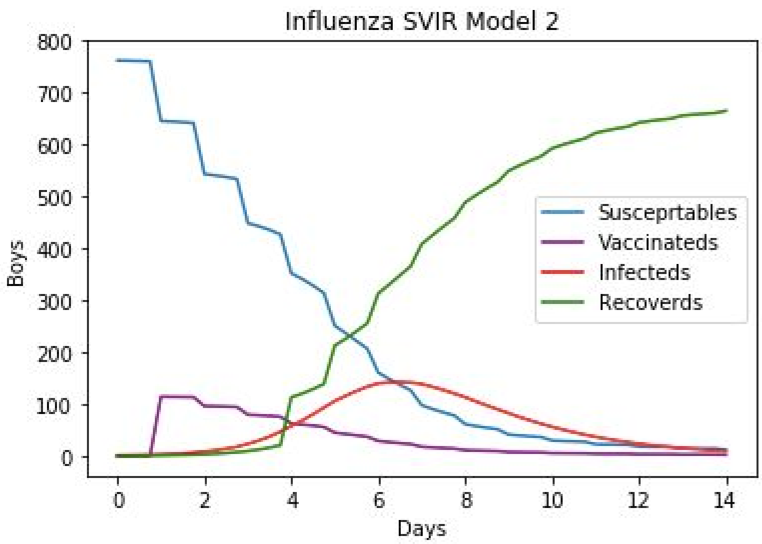
This section of the code is very similar to the first. This section accounts for the days before the first vaccinations start to kick in. Everything is calculated in the same way, the only thing is that the number of recovered boys does not include the boys who were vaccinated two day ago because it hasn't been that long yet.

```
else:
    vaccinated_today = vaccinated_today - (infection_rate * vaccinated_today * infecteds)*DT
    s1 = (infection_rate * vaccinated_today * infecteds)
    vaccinated_yesterday = vaccinated_yesterday - (infection_rate * vaccinated_yesterday * infecteds)*DT
    s2 = (infection_rate * vaccinated_yesterday * infecteds)
    vaccinated_twodaysago = vaccinated_twodaysago - (infection_rate * vaccinated_twodaysago * infecteds)*DT
    s3 = (infection_rate * vaccinated_twodaysago * infecteds)
    susceptibles = susceptibles + (-get_sick) * DT
    infecteds = infecteds + ((get_sick+s1+s2+s3) - recover) * DT
    recovereds = recovereds + (recover) * DT
```

This final part of the code updates each variable for all the other times(t) that aren't 12:00am on that day. These formulas update every 6 hours (Δt). The number of vaccinated boys all update to account for the boys who got sick in those last 6 hours. The number of susceptibles and the number of recovereds update like the original SIR model. The infecteds includes the number of boys who were vaccinated, but still got sick in the last 6 hours.

	Days	Susceprtables	Vaccinateds	Infecteds	Recoverds
0	0.0	762.000000	0.000000	1.000000	0.000000
4	1.0	645.415270	113.920362	2.901494	0.762718
8	2.0	543.017279	95.883944	8.298212	2.969808
16	4.0	351.883486	62.385066	57.583307	112.823900
24	6.0	160.464987	28.714541	139.622356	312.689668
32	8.0	61.100125	10.917412	112.980715	488.379960
40	10.0	29.541380	5.245477	55.993300	592.628569
48	12.0	17.794561	3.148283	23.444727	642.364761
56	14.0	11.947245	2.110440	9.137233	665.008451



In this graph we can see how this epidemic differs from the others. The number of infected boys isn't close to the number of boys with the flu in the original model, which is to be expected when introducing a vaccine into the environment. It’s still larger than the number of infected boys in model 1, which is due to the 3 day delay in the effectiveness of the vaccination. Since there are more boys that are still able to catch the disease, there are more boys who are likely to catch the disease.

(3) In the third simulation, the adults were prepared for flu season and administered flu shots to all 762 boys two days before the one boy gets the flu. Unfortunately, the shot doesn’t start working until four days after the vaccination. This means that the boys have a chance of catching the flu for two days of the simulation.

```
if t == 2: #checks if it is the second day
    susceptibles = susceptibles + (-get_sick) * DT - vaccinated_boys #calculates the number of susceptible boys subtr
    infecteds = infecteds + (get_sick - recover) * DT #calculates the number of infected boys
    recovereds = recovereds + recover * DT + vaccinated_boys #calculates the number of recovered boys adding the boys
elif t < 2:
    susceptibles = susceptibles + (-get_sick) * DT #calculates the number os susceptible boys
    infecteds = infecteds + (get_sick - recover) * DT #calculates the number of infected boys
    recovereds = recovereds + (recover) * DT #calculates the number of recovered boys
    vaccinated_boys = vaccinated_boys - (infection_rate * vaccinated_boys * infecteds) #calculates the number of vacc
else:
    susceptibles = susceptibles + (-get_sick) * DT
    infecteds = infecteds + (get_sick - recover) * DT
    recovereds = recovereds + (recover) * DT

get_sick = infection_rate * susceptibles * infecteds
recover = recovery_rate * infecteds
```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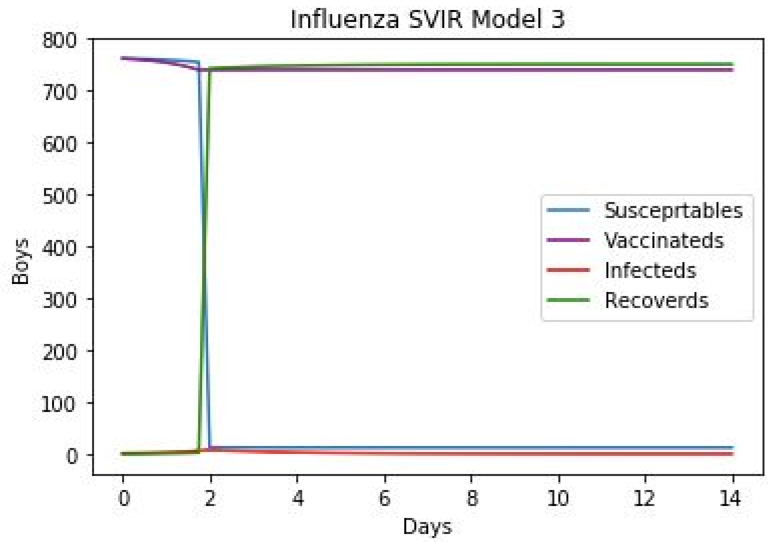
The only change to the initial statements is that the number of vaccinated boys (vaccinated_boys) starts at 762. We then introduce an **if** statement that checks to see if it is day two. If it is day two, then we subtract the number of vaccinated boys from the susceptible boys, since the vaccination kicks in on day two. Then we calculate the infecteds normally and add the number of vaccinated boys to the recovereds. All the vaccinated boys are considered recovered on the 2nd day. If t < 2, then the number of susceptibles, infecteds, and recovereds calculates normally, and the number of vaccinated boys is affected by the infecteds (vaccinated_boys - (infection_rate * vaccinated_boys * infecteds), since some of the vaccinated boys get infected during the two days before the vaccine becomes effective. Otherwise, t > 2 and the model runs like the original SIR model.

	Days	Susceprtables	Vaccinateds	Infecteds	Recoverds
0	0.0	762.000000	762.000000	1.000000	0.000000
4	1.0	759.469082	754.130056	2.768200	0.762718
8	2.0	12.470205	740.047549	7.613190	742.916605
16	4.0	12.195186	740.047549	2.781027	748.023787
24	6.0	12.096377	740.047549	1.014963	749.888660
32	8.0	12.060533	740.047549	0.370300	750.569167
40	10.0	12.047484	740.047549	0.135084	750.817431
48	12.0	12.042728	740.047549	0.049276	750.907996
56	14.0	12.040993	740.047549	0.017975	750.941032



We start with hardly any change in the susceptibles and the vaccinateds, which both start at 762, and a similar change in the infecteds and recovereds, which both start at zero. As expected, the largest effect on the graph happens on day 2, which is when the vaccine starts working. In our simulation, the vaccine starts working immediately on day 2, so there is a dramatic drop in the number of susceptible boys and an incredible increase in the number of recovered boys. Because the vaccine starts working too quickly, there isn't any time for the flu to really spread. We only have a few unlucky boys who manage to get infected in the first 2 days of the simulation. We do have a remainder of 12 susceptible boys who will probably stay susceptible since the number of infected boys is less than one, so there is little to no chance of these boys catching the flu.

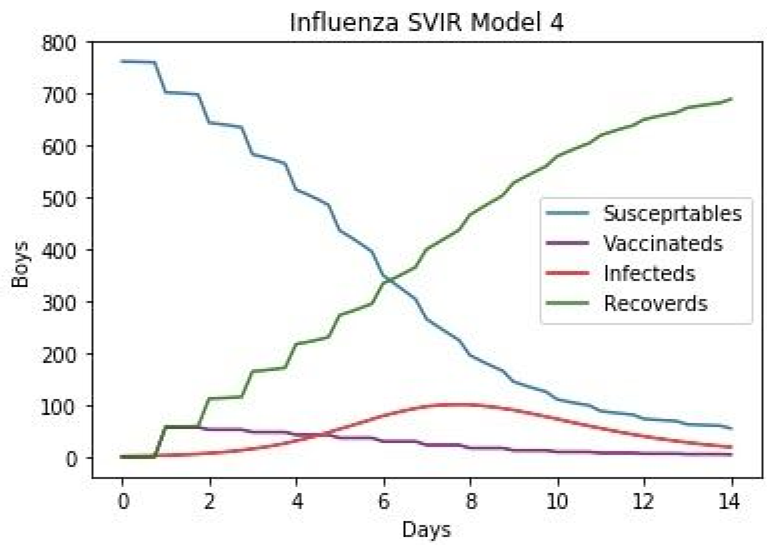
Further Analysis:

To provide further analysis on the effect of changing the vaccine parameter in the SVIR simulation, we looked precisely at the impact that changing the number of susceptible boys vaccinated each day from 15% to 7.5% and 30% in first project question would have on the spread of the disease and thus the final number of susceptibles, infecteds and recoverds at the end of the 14 days (56 time steps). Moreover, we looked at the effects of changing the time it takes for the vaccination to work from 4 days to 6 days on the spread of the flu when all the boys are vaccinated 2 days before the start of the simulation as in question 3.

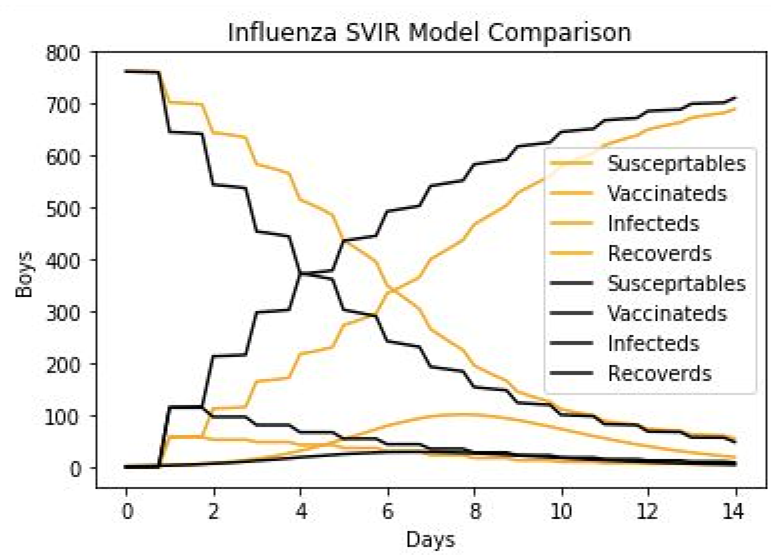
In response to the first part of the first exploratory question, we used the same code that was used to arrive at the results for question 1, slightly modified to use a vaccination percentage of 7.5% (the original percentage divided by 2). The following table shows the results of this modification every two days:

	Days	Susceprtables	Vaccinateds	Infecteds	Recoverds
0	0.0	762.000000	0.000000	1.000000	0.000000
4	1.0	702.442176	57.026906	2.768200	57.789624
8	2.0	643.909000	52.378593	6.906727	112.184274
16	4.0	514.885166	42.399018	30.858387	217.256447
24	6.0	349.973140	29.640989	78.968285	334.058575
32	8.0	195.840200	16.881427	100.555176	466.604624
40	10.0	110.861280	9.418578	73.074804	579.063916
48	12.0	73.144777	6.085494	39.695621	650.159602
56	14.0	54.847498	4.502134	18.834485	689.318017

As expected, decreasing the percentage of boys that are vaccinated each day negatively affects the total number of susceptible, infected and recovered boys at the end of the 14 day simulation. Surprisingly however, this effect is not significantly large. Despite having reduced the number of boys that are vaccinated by half, the numerical difference by the end of the simulation is not at all drastic. For example, the number of recovereds by the end of this simulation is only about 21 less than in the one where 15% were vaccinated (as in question 1). Similarly, the difference in susceptibles is only about 7, while 15 in infecteds. This may indicate that since the number of boys that were vaccinated each day was already so small at 15%, decreasing it wouldn't have a significant impact on the overall spread of the flu. Moreover, the total number of boys without the flu at the end of the simulation goes from about 757 to approximately 744, a difference of 15 boys. This shows (as well as the difference in infecteds) that 15 more boys would not have the flu at the end of the simulation if the percentage of boys that are vaccinated was twice as large (15%). The following graph models this outcome:



Due to the fact that the behaviors of the variables in this model are almost indistinguishable from the ones observed in the graph corresponding to question 1, the following graph was made to clearly see the difference between this change in vaccination percentage:



This graph shows the change between the original effect of the number of boys vaccinated each day (15% as in question 1) on the spread of the flu (shown in black) and the effect of the lowered vaccination percentage of 7.5% (shown in orange). This better shows that despite the resulting populations at the end of the simulation being very close together, the gap is still quite large during the simulation. Moreover, the number of boys that get infected during this time is much larger (when less boys are vaccinated each day).

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In response to the second part of the first exploratory question, we used the same code that was used to arrive at the results for questions 1 (in the project questions) and the first part of the first exploratory question, slightly modified to use a vaccination percentage of 30% (the original percentage multiplied by 2). The following table shows the results of this modification every two days:

	Days	Susceprtables	Vaccinateds	Infecteds	Recoverds
0	0.0	762.000000	0.000000	1.000000	0.000000
4	1.0	531.361458	228.107624	2.768200	228.870342
8	2.0	368.723024	158.563297	5.079305	389.197671
16	4.0	175.217201	75.507999	7.143654	580.639145
24	6.0	83.293446	35.844000	4.945586	674.760968
32	8.0	40.093350	17.218265	2.393257	720.513393
40	10.0	19.489492	8.359748	0.972188	742.538320
48	12.0	9.520023	4.081316	0.362597	753.117380
56	14.0	4.659466	1.997143	0.129702	758.210833

As can be observed, the difference here is much more drastic. Doubling the number of boys that are vaccinated each day has a much greater effect on the total number of susceptible, infected and recovered boys at the end of the 14 day simulation than dividing the number of boys vaccinated by 2. For example, the number of recovereds by the end of this simulation is 758, about 47 more than in the one where 15% were vaccinated (as in question 1). Additionally, there are about 43 less succeptables and 4 less infecteds in this simulation. Precisely, there are no infecteds (when rounded) in this simulation. That is, doubling the number of boys that are vaccinated each day results in having no boys with the flu at the end of the simulation. This shows that the immunizations can be very effective in reducing the spread of the disease when administered to a percentage of the population that is large enough (above 30%). The following graph visually represents the spread of the flu when 30% of susceptible boys each day:

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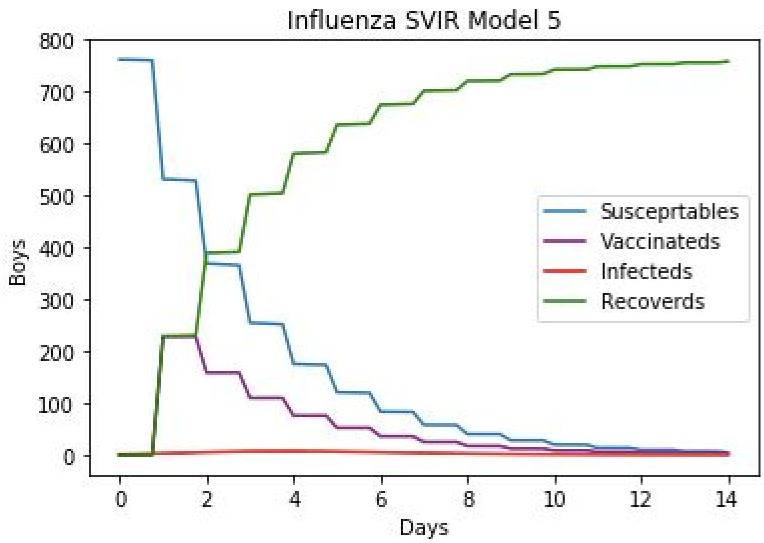
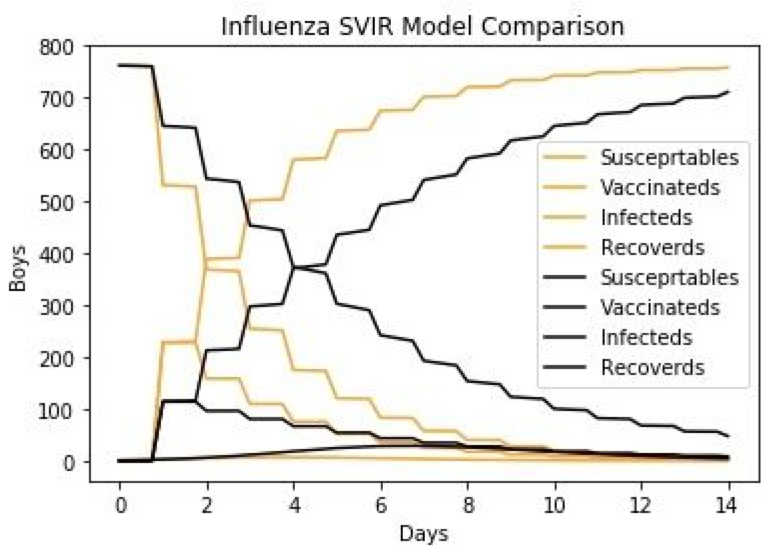


As in the first part of the first exploratory question, since the behavior of these functions is too similar to the behavior of the ones observed in the graph corresponding to question 1, the following graph was made to see the difference between this change in vaccination percentage more easily:



This graph shows the change between the original effect of the number of susceptible boys vaccinated each day on the spread of the flu (15% vaccination percentage as in question 1) which is shown in black, and the effect of the doubled vaccination percentage of 30% on the spread of the flu (shown in orange). Both graphs display a very similar pattern. The one with an increased vaccination percentage however, is shifted to the left because boys recover and move away from

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being susceptible much earlier on in the simulation due to the larger amount of boys being vaccinated. Moreover, this simulation's graph is wider showing a faster rate of recovery and rate at which boys become no longer susceptible.

In response to the second exploratory question, we used the same code that was used to arrive at the results for question 3, slightly modified so the vaccination becomes effective after 8 days instead of 4. This means that the code needs to focus on day 6 instead of day 2.
The following table shows the results of this modification every two days:

	Days	Susceprtables	Vaccinateds	Infecteds	Recoverds
0	0.0	762.000000	762.000000	1.000000	0.000000
4	1.0	759.469082	749.008435	2.768200	0.762718
8	2.0	752.517754	714.190306	7.613190	2.869056
16	4.0	686.099797	440.042531	52.995515	23.904688
24	6.0	360.721513	44.289303	217.729193	184.549294
32	8.0	117.196794	44.289303	220.735248	425.067958
40	10.0	53.274003	44.289303	112.761274	596.964723
48	12.0	37.171152	44.289303	48.250610	677.578238
56	14.0	32.041044	44.289303	19.652324	711.306632

As we can observe, the end result is pretty similar when we look at the susceptibles, infecteds, and the recovereds. The drastic difference between the two models is that the number of infecteds in this model increases exponentially between the 2nd and 7th days. At one point, the number of infected boys in this model reaches over 200 while in Model 3 the number of infected boys only reaches 7. Another drastic difference between the two models is the number of boys vaccinated each day. In Model 3, the boys who receive a vaccination don't have time to come into contact with enough infected boys for there to be any real change. In this model the boys who were vaccinated are getting sick just like they would if they weren't vaccinated. By the time the vaccination starts working, there aren't a lot of boys left to impact the number of recovereds. In this case, the vaccination doesn't have enough impact on the original SIR model for the vaccination to be worth it.

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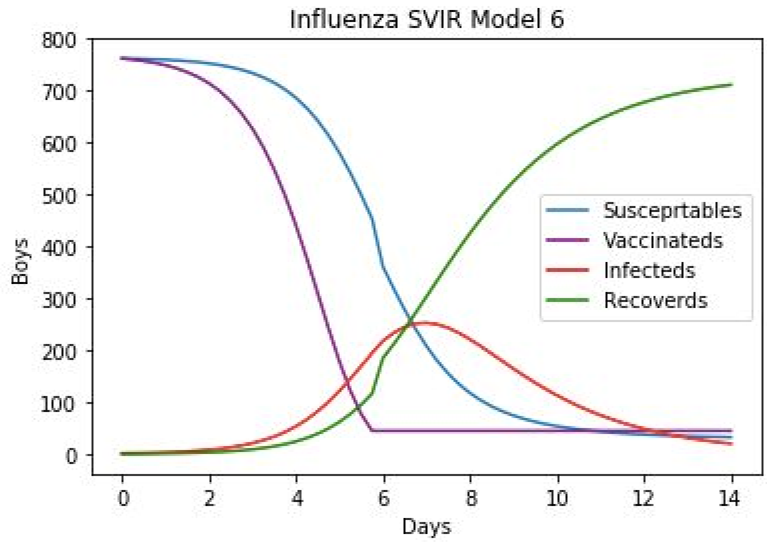
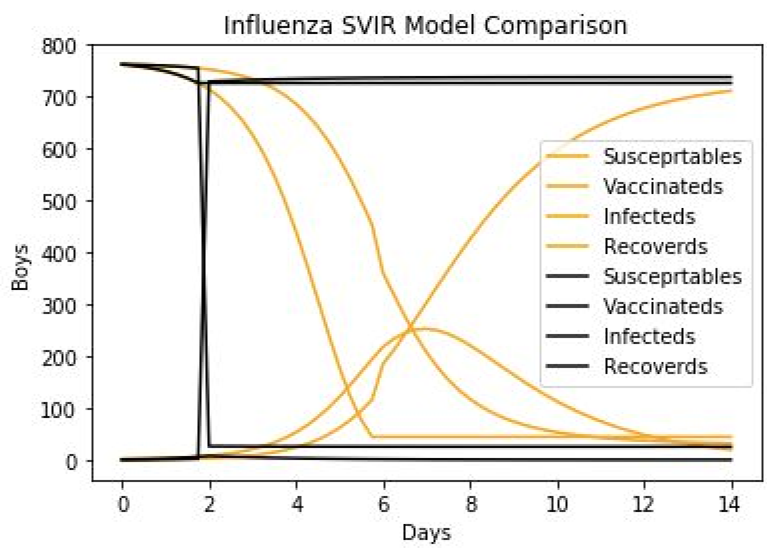


In the following graph, we can really see the impact that the delay in the effectiveness of the vaccine has on the spread of the disease.



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To compare the final results of every simulation created above and predict the most effective measures that can be taken during an Influenza epidemic, the following table was created:

	Susceptibles	Infecteds	Recovereds
No Vaccine	29	21	713
Model 1	48	4	711
Model 2	12	9	665
Model 3	25	0	738
Model 4	55	19	689
Model 5	5	0	758
Model 6	32	20	711

Based on these rounded results, it is evident that Model 3 is the most effective way to prevent the spread of disease (exact values show that Model 3 had 0.026686 infected while Model 5 had 0.129702), where Model 3 shows the simulation in which all boys are vaccinated 2 days before the simulation and the vaccines become effective 4 days after (2 days into the simulation). We also see that Model 3 is more effective than Model 5 by looking at the number of recovered boys. Model 3 has a smaller number of recovered boys, which implies that there was a smaller number of infected boys during the simulation who needed to recover. This may be due to the fact that everyone was vaccinated prior to the introduction of the flu into this environment and the time it takes for the vaccine to start working is much smaller. Despite not being as effective as Model 3, Model 5 (simulation in which 30% of susceptible boys are vaccinated each day with a vaccination that is effective immediately) is also promising in reducing the spread of the flu. However, this is a less realistic measure to opt for as it isn't likely that a vaccine will be effective right away. That said, vaccinating the whole population as soon as possible is not only more feasible, but slightly more effective.

What Could We Have Done Differently?:

The main thing we could have done differently was introduce a vaccination rate. Instead of continuing the kind of behavior in the first model (immediate effectiveness), we could have gradually introduced the vaccination into the environment. There wouldn't be such a drastic drop in the number of susceptible boys in model 3, but a gradual decrease as the vaccination got more and more effective as it approached the fourth day. This kind of behavior would be similar in model 2. The graph would look similar, but we predict that the susceptibles and the recovereds would increase and decrease at a more constant rate, before tapering off towards zero and 762. We could have introduced a quarantine after a certain number of boys were confirmed to have

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gotten infected. This would decrease the number of boys who get sick in the long run. We would expect that the number of susceptibles would reach a constant that was above zero and the number of recovereds would never reach the total population since some of the boys were protected because of the quarantine.

Conclusion:

This project allowed for the study of a simulation of the spread on Influenza through a closed environment given the previously mentioned constraints. Using an SIR model, allowed for the prediction of the number of boys who would be infected with the disease, who would be recovered from the disease, and who would be still vulnerable to the disease at a given time. Python was used to display this SIR model as well as SVIR models that formed a large part of the project's analysis. In each model, we were able to see how different versions of a flu vaccine had an impact on the overall spread of the epidemic. As mentioned previously, the most difficult model was the vaccination with a 3 day delay. We understood the overall concept of what we were trying to achieve, but we struggled with how to transfer that knowledge into Python. The resulting code we came up with fits the idea and we believe it to be correct. Unfortunately, we struggled with this model for so long that we failed to look into any further analysis of the problem. We did predict how introducing a quarantine could impact the spread of the epidemic in the previous section. Overall, we would recommend that the school doctors find a fast acting vaccine (preferably one that starts working immediately) and vaccinate as many boys that they can at a given time in order to reduce the number of boys who get infected. If they are unable to do this, then we recommend implementing a quarantine to reduce the amount of contact between the boys and advise everyone to wash their hands and cover their mouths when they sneeze and cough.

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