Feral Cat Control Model

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Introduction:

The following paper provides a detailed outline of an explorative mathematical modeling project. The goal of this group project was to explore the impact of the feral cat population when left unattended and to analyze the methods to control this population. We are given the following simulation:

We were recently approached by Mr. Tom Kat, a representative from a town that has a very large feral cat colony. He offered us a wonderful opportunity. Mr. Kat is willing to fund a project where we implement humane methods for population control in his town. We will also have the opportunity to give public presentations nationwide on the proper protocol for controlling feral cat populations.

Before officially promising us the funds we need, Mr. Kat has asked that we produce a formal report utilizing simulated mathematical data to illustrate the efficacy of our suggestions. If he likes the results of these simulations, then we can begin our new partnership. There are two commonly used methods for controlling feral cat populations. In the first method, traps are set for feral cats and when a cat is trapped it is immediately euthanized. This is called the "trap-kill" method. We do not believe this method is ethical, and therefore we advocate for a different method, which is commonly called "trap-neuter-return." In this method traps are set, but when a cat is trapped it is neutered/spayed and returned to the population. We are particularly interested in the relative effectiveness of these two methods at controlling the overall cat population.

Thus, we will use mathematical approaches, particularly differential equations, to model the general behavior of the feral cat population in each of the following cases:

1. No Control: we set no traps,
2. Trap-Euthanize: we set traps and kill the cats we catch,
3. Trap-Neuter-Release: we set traps, neuter/spay the cats we catch, and release them back into their environment.

The goal of this project is not only to model each of these scenarios, but to show the drastic change in feral cat population over a period of time when control measures are taken. Moreover, we sought to compare the overall effectiveness of each population control method (2) and (3), and determine the specific number of traps that will yield optimal results.

Givens:

Across all models we assume the following:

- $0 \leq t \leq 10$
- $P_0 = P(0) = 100$
- $k = 1.08$
- $C = 12$

Where:

- t is time, and we observe the change in population over 10 years
- P_0 is the initial population, and the population at time t = 0 is P(t) = P(0) = 100
- k is the (growth rate) constant of proportionality, or the number of generations per year
- C is the number of cats each trap catches per year
- T is the number of chosen traps which remains the same across all models but is changed to observe the relative effectiveness of the number of traps used

Mathematics Used:

For this project, we are concerned with a specific set of models or equations known as "population growth models", as they pertain to the growth of populations. They are differential equations in that they include a function and its derivative, or rate of change. In this case we use P(t), the (feral cat) population function, which takes time t as an input, and outputs the total population. In other words, for any t that is fed into the function, we obtain the population at time t (which is a mere prediction). As P takes t as an input, we say that population is a function of time, meaning that it changes with respect to time. We also use the derivative of this function dP/dt, which yields the rate of change of the population. That is, it gives the rate at which the population is growing (or will grow). Thus, if the rate of change is 0, then it is implied that the population is constant and not growing at any rate. Similarly, if it is positive, we know that the population is increasing (or changing at a positive rate), and if it is negative, that it is decreasing (or changing at a negative rate). It is often the case that the derivative of a function is also a function of time. However, in the case of differential equations in the context of population growth, the derivatives of population functions are themselves functions of population. That is, the rate at which a given population grows at a given time is a function of the population at that time. In our case, the rate of change of the population dP/dt, is actually proportional to the current population P. Specifically, dP/dt is equal to some constant k times P, where k is called the proportionality constant. When we put all of this together, we get the following differential equation:

$$\frac{dP}{dt} = kP$$

The information we are initially given allows us to derive this equation. Namely, that the population grows (or changes) at a (positive) rate proportional to the current population. However, since we don't know what P(t) actually is, other than what it will tell us, we can find it by solving the differential equation. That is, by deriving some P that will satisfy the differential equation. In other words, finding a function whose derivative is a constant times itself. Doing this requires an understanding of calculus and the formal process of integration. However, for our purposes, it suffices to say that the only function which has this property is the exponential function $y(x) = e^x$. Calculus tells us that the derivative (often written as dy/dx) of e^x is just itself e^x. Moreover, anything of the form $e^x + c$, where c is an arbitrary constant, also has a derivative of e^x , as the derivative of any constant is 0. We call the function that we differentiate the antiderivative. Thus, there are an infinite number of values c can take and a corresponding infinite number of functions whose derivatives are all e^x , we say that e^x has an infinite number of possible antiderivatives, which we generalize by writing $e^x + c$. This knowledge allows us to derive the following general solution to the differential equation:

$$P(t) = Ae^{kt} \qquad \text{where A is a constant}$$

To obtain a particular solution which satisfies the initial condition we were given, stating that the initial population (at time t = 0) must be equal to 100, we substitute 0 for t and 100 for P to obtain a value for A. After doing so, we see that $c = P_0 = 100$ satisfies the initial condition, and we can write the particular (or case-specific) solution to the differential equation as follows:

$$P(t) = P_0e^{kt} = 100e^{kt}$$

The differential equation mentioned above and corresponding particular solution make up the general model for the first scenario we are observing. Namely, the behavior of the feral cat population without using a control method. With this model, we are able to see how the population will naturally change over time without intervention. We use the results obtained in this model to observe the overall impact of the two population control methods on the feral cat population. To derive the models for these control methods, we use a similar approach and adjust our current differential equations accordingly as follows:

Trap-Kill Method:

$$dP/dt = kP - CT$$

$$P(t) = (Ae^{kt} + CT) / k \qquad \text{where } A = 108 - 12T$$

Trap-Neuter-Return Method:

$$dP/dt = kP - CTt$$

$$P(t) = CTt + \frac{CT}{k} + Ae^{kt} \qquad \text{where } A = 100 - \frac{CT}{k}$$

Interpreting the Models and Solutions:

In order to analyze the change in population over time, we need to calculate the solutions to the differential equations introduced in the previous section.

In order to solve the differential equation representing the No Control method, $\frac{dP}{dt} = 1.08P$ when $P_0 = 100$, we must find a function P(t) whose derivative is the product of k with P(t). We know from Section 1.1 of "Differential Equations" (Blanchard), that we can use the solution $P(t) = e^{kt}$. This would give us the solution,

$$P(t) = 100e^{1.08t}.$$

This is a positive exponential equation, so it will constantly be increasing. In this case, the cats have no restraints when it comes to reproduction, and their environment is able to support the population, no matter the size. This is not exactly realistic, but it portrays the idea that if we do nothing, the population will continue to increase, ultimately impacting the environment in a negative way.

In order to solve the differential equation representing the Trap-Euthanize method, $\frac{dP}{dt} = 1.08P - 12T$ where $P_0 = 100$ and T is the number of traps, we have to use the separating variables method.

First we substitute, $P'(t) = 1.08P(t) - 12T$.

Then rewrite in the form of a first order separable ODE, $\frac{1}{1.08P(t) - 12T} P'(t) = 1$.

Then we solve an get $\frac{ln(1.08P(t) - 12T)}{1.08} = t + c_1$.

And we isolate P(t) to get $P(t) = \frac{e^{1.08t + 1.08c}}{1.08} + \frac{12T}{1.08}$.

Simplified to the solution,

$$P(t) = \frac{1}{1.08}(Ae^{1.08t} + 12T), \text{ where } A = 108 - 12T.$$

The population of feral cats in this case depends on the number of traps. We take the total population of cats and then subtract the number of cats we caught that year to get the new population.

In order to solve the differential equation representing the Trap-Neuter-Release method, $\frac{dP}{dt}$ = 1.08P - 12Tt where P_0 = 100 and T is the number of traps, we use integrating factor to get,

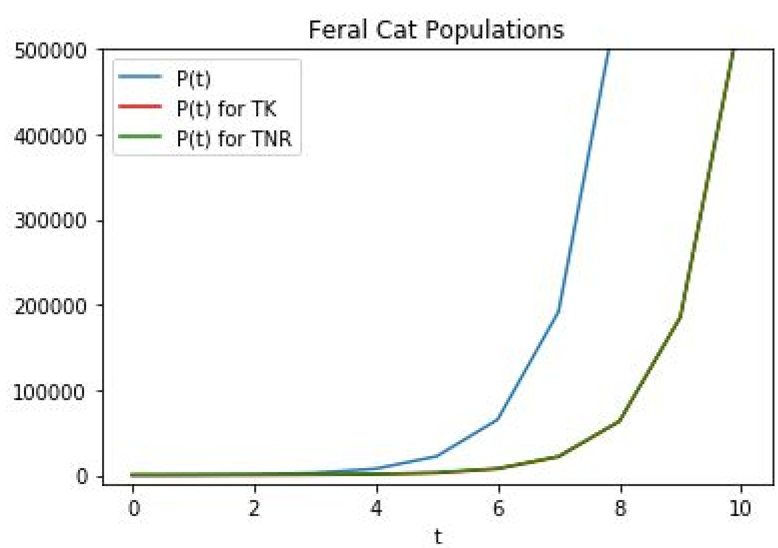
$$P(t) = 12Tt + \frac{12T}{1.08} + Ae^{1.08t}, \text{ where } A = 100 - \frac{12T}{1.08}.$$

The population of feral cats depends on the number of traps again in this case, but it also depends on time t. Because the number of unsterilized cats is decreasing at each time t, we have to account for time when calculating this model. In the solution we can see the number of sterilized cats now being added back into the total population.

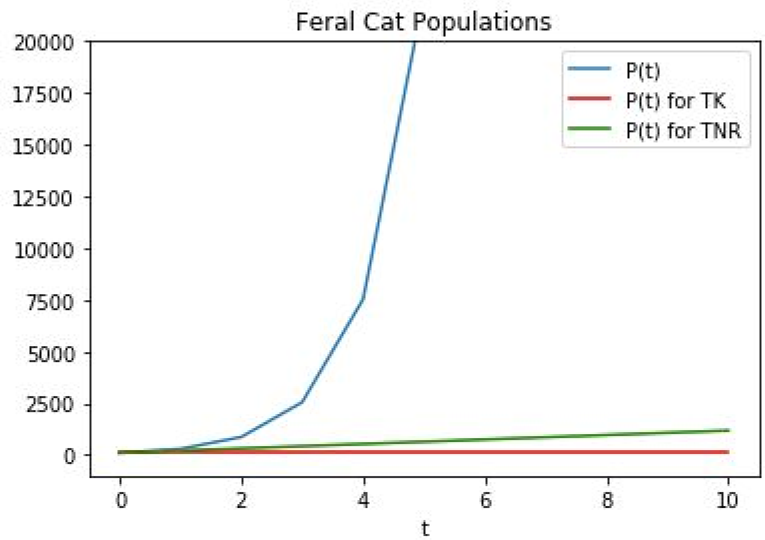
Exploration:

As we change the number of traps used, the population of cats is hindered in a negative manner. If we weren't to use traps, we would have an endless exponential increase in the population of cats. But we *are* going to use traps, and we found that when the number of traps is less than 9 then the population is impacted in a drastically different way compared to when the number of traps is greater than 9 or equal to 9. This means that the critical number of traps is 9.

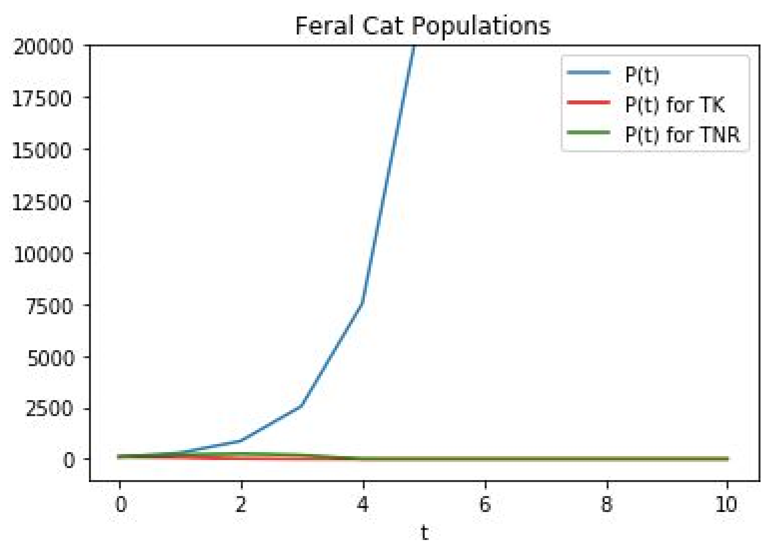
So, when T < 9, the number of cats that are being caught in the Trap-Euthanize and the Trap-Neuter-Release methods isn't enough to get the population under control and the exponential increase between the two is basically the same, as shown in the graph. (We promise we graphed the P(t) for Trap-Kill, it just overlaps with the TNR method.)



When T = 9, the number of cats that are being caught in the Trap-Euthanize method is just enough to keep the population at 100 cats. In the Trap-Neuter-Release method, the population is increasing, but at a slower rate, as shown in the graph below:



When T > 9, the population of cats decreases exponentially in the Trap-Euthanize method immediately and promptly reaches 0. In the Trap-Neuter-Release method, the population increases at first and then decreases to 0, as shown in the graph below:



So, in the Trap-Euthanize method, it is important to have more than 9 traps, otherwise this method continues to increase rapidly or is constant at 100 cats. In the Trap-Neuter-Release method, it is important to have more than 8 traps. This decreases the number of cats significantly and if there are more than 9 traps, the population reaches 0.

Comparing the Models:
In the long-term behavior, the No Control method is ineffective in controlling the feral cat population because realistically, if there is no restriction imposed on the cats' reproduction system and there are enough resources for them, the cat population will always be increasing. Meanwhile, the Trap-Euthanize method and the Trap-Neuter-Release method yield positive results because we will be putting restraints on the reproduction of the cats by setting up traps to either euthanize them or neuter them, thereby slowing the growth rate of the feral cat population. Due to this result, it comes down to whether the Trap-Euthanize method or the Trap-Neuter-Release method is more effective.

As mentioned in the previous section, the number of traps set to catch the feral cats are essential to limiting the population growth, and we have determined the critical threshold to be at 9 traps for both the Trap-Euthanize and Trap-Neuter-Release methods. In respect of the three cases where there are fewer than 9 traps, exactly 9 traps, and more than 9 traps, we will be comparing the cat population after 10 years when applying the Euthanize approach and the Neuter approach when the number of traps is 8, 9, or 10.

When there are 8 traps, we get the following data:

	t	P(t)	P(t) for TK	P(t) for TNR
0	0	100	100	100
1	1	294	122	218
2	2	867	185	377
3	3	2553	373	661
4	4	7519	924	1308
5	5	22141	2549	3029
6	6	65197	7333	7909
7	7	191985	21421	22093
8	8	565333	62904	63672
9	9	1664724	185058	185922
10	10	4902080	544764	545724

The population of feral cats over 10 years still increases drastically when using either the Trap-Euthanize method or the Trap-Neuter-Release method. This means that it may be more effective if more traps are set.

When there are 9 traps, we get the following data:

	t	P(t)	P(t) for TK	P(t) for TNR
0	0	100	100	100
1	1	294	100	208
2	2	867	100	316
3	3	2553	100	424
4	4	7519	100	532
5	5	22141	100	640
6	6	65197	100	748
7	7	191985	100	856
8	8	565333	100	964
9	9	1664724	100	1072
10	10	4902080	100	1180

While the population for the Trap-Euthanize method remains constant at 100 cats during the ten-year period, the population for the Trap-Neuter-Release method still increases after 10 years but at a much slower rate compared to the case in which 8 traps are set.

When there are 10 traps, we get the following data:

	t	P(t)	P(t) for TK	P(t) for TNR
0	0	100	100	100
1	1	294	78	198
2	2	867	15	255
3	3	2553	0	187
4	4	7519	0	0
5	5	22141	0	0
6	6	65197	0	0
7	7	191985	0	0
8	8	565333	0	0
9	9	1664724	0	0
10	10	4902080	0	0

If there are 10 traps set per year, the population will decrease for both the Trap-Euthanize method and the Trap-Neuter-Release method. For the Euthanize method, the number of cats will be drastically decreasing and the cat colony will disappear after 3 years. For the Neuter method, the population fluctuates slightly for a few years and eventually gets to 0 after 4 years.

From the three cases above, it can be concluded that the Trap-Neuter-Release method is the most efficient and humane method for controlling the feral cat population since the population can be constrained naturally without having to euthanize any of the cats. In order for the control to succeed, the number of traps has to be greater than 9.

Conclusion:

The purpose of this paper is to analyze different approaches to controlling the feral cat population in Mr. Kat's town, and our goal is to determine which of the proposed methods is the best one to implement and worth the funds that Mr. Kat has offered. After rigorous research and analysis, we have come to the conclusion that among the No Control method, the Trap-Euthanize method, and the Trap-Neuter-Release method, the Trap-Neuter method proves to be the most effective and is able to help limit the growth rate of feral cats in a short amount of time (within three to four years) with a reasonable number of traps. It is also the most humane approach as the cats do not have to be permanently removed from the colony. Therefore, based on the results, we suggest that Mr. Kat set 10 traps per year in order to spay the caught feral cats and return them to the colony. It goes without saying that there are other factors that we did not take into account while constructing the predictive models. For example, we did not consider the relationships between feral cats and other species and humans or the surrounding environment, which may support or hinder the effectiveness of the Trap-Neuter-Release method. But for this general model about the population of feral cats, we recommend this method to help stabilize the feral cat population.